## V.—CRITICAL NOTICES.

*The Principle of Relativity, with Applications to Physical Science.* By A. N. Whitehead. Cambridge University Press. Pp. xii, 190.

Prof. Whitehead's two former works on the philosophy of Nature, which have been reviewed in Mind by Prof. Taylor and the present writer, gave a highly original and profoundly important theory which led up to the well-known transformation equations of the Special Theory of Relativity. They did not, however, deal with the General Theory and the modifications involved thereby in the Law of Gravitation. In the present work Prof. Whitehead treats the General Theory from his own standpoint, which differs fundamentally in certain respects from that of Einstein or Weyl. The book falls into three parts. The first is mainly epistemological; the second consists of a number of detailed applications of Prof. Whitehead's formula for the law of gravitation to certain physical phenomena; the third is purely mathematical, and is a sketch of the general theory of Tensors.

Parts II. and III. are too technical for detailed review in Mind, even if I were competent to do this adequately. I will confine myself to saying that in Part II. Prof. Whitehead shows that his law of gravitation leads to all the results which have been deduced from Einstein's law and verified. He further deduces a number of consequences in electricity, magnetism, heat, etc., by which his own law might be experimentally tested; but, unfortunately, the effects thus deduced are probably all below the limits of our present powers of observation. He has also applied his law to the problem of the Moon's motion, and has reached certain modifications in the results of the classical theory; but apparently the observations needed to verify or refute these consequences are at present lacking. Part III. makes certain improvements in the usual notation for tensors, and has the great advantage of treating the whole subject in a perfectly general way without putting geometrical interpretations on the various special tensors which are introduced. So far as I can judge it seems well fitted to conduct anyone with a fair knowledge of mathematics into the heart of the subject.

The only criticism which I have to make on these parts of the book is that they seem to me to consist too much of isolated snippets, and that they could be improved by a good deal more explanation, illustration, and general "connective tissue". I should not have ventured to make these criticisms had I not found that several of my mathematical colleagues shared my difficulties to some extent. I wholly disagree with Whitehead's admiration for the practice of writing books in the lecture form. After lectures people can ask questions, and their difficulties can often be cleared up in a few words. Readers of books are not in this happy position; and therefore admirable lectures may make very bad books, if published without supplement or modification. I am quite certain that Prof. Whitehead would have written a book which would have been far better understood and would have exercised a far wider influence if he had not simply reproduced his various lectures, but had used them as the basis for a larger, more detailed, and more coherent work. It is the more regrettable that he has not done this, because his main thesis is so important and original that it were a thousand pities if it should be ignored by those whom it vitally concerns, on account of defects in his exposition. Having said so much about Parts II. and III. I will now confine myself to the general view developed in Part I., which forms the basis of the whole and will be of most interest to the majority of readers of MIND.

The points of agreement and the points of difference between Whitehead and other Relativists, like Einstein, can be stated quite shortly and simply. He agrees with them that the fundamental relations in Nature are not spatial or temporal but are spatio-temporal, and that Space and Time are two abstractions from Space-Time, just as planes and straight lines are two abstractions from volumes. The fundamental stuff of Nature is therefore events, which have duration as well as extension. Again, he agrees that there is not just one possible way of slicing up Space-Time, leading to one unique Space and one unique Time. There is an infinite plurality of different "directions" in Space-Time, each of which is an equally permissible time-axis for dating all physical events. Corresponding to each of these there is a certain timeless space; and all these timeless spaces are equally permissible for placing all physical events, provided you use the corresponding time-direction for dating them. Thirdly, he agrees that not *all* directions in Space-Time are permissible time-axes; all those which are permissible as time-axes are confined within a certain four-dimensional cone. Next, he agrees that the fact that the fundamental relations in Nature are spatio-temporal necessitates a modification in the traditional law of gravitation. This law takes account only of the *distances* between *bodies*, whereas a genuine law of Nature must be in terms of the *spatio-temporal* separation between *events*. He agrees, moreover, that the laws of Nature must be expressible in tensor form, and holds that Einstein's law, when properly interpreted, is at least a *possible* form of the law. These, I think, are the main points of agreement between Whitehead and the orthodox Relativists. We must now consider the points of difference.

In the first place, Whitehead's method of reaching the transformation equations of the Special Theory of Relativity is different from the classical method. The classical deduction starts from the empirical fact that the measured velocity of light with respect to different sets of material axes is the same, in spite of these axes being in motion with respect to each other, provided that they are resting or moving uniformly in straight lines with respect to a Newtonian frame of reference. The grounds for accepting this as a fact are the negative results of the Michelson-Morley experiment and of certain electro-magnetic experiments. Whitehead's method of treatment has been fully developed in his earlier works and has been described by me in MIND in my review of his *Principles of Natural Knowledge*. A summary is given in the present book, but I do not think it could possibly be understood by anyone who was unacquainted with the earlier writings. Whitehead starts from the conception of a plurality of different time-systems, such that durations belonging to any one are parallel and durations belonging to any two intersect each other. He is then able to define the momentary spaces of any time-system and to define the parallelism of planes and lines in any momentary space. Next, by considering motion and rest, he is able to define the timeless space of any time-system, and to show that its geometry is the same as that of the momentary spaces, although its points, straight lines, and planes, are different from those of momentary spaces and of Space-Time. The conception of a time-system also enables him to define normality. Having defined parallelism and normality he is able to set up a system of measurement for Space-Time. He takes it as axiomatic (*a*) that the opposite sides of parallelograms are congruent. This enables him to deal with the congruence of stretches on different but mutually parallel straight lines. (*b*) His second axiom is that if a normal be drawn from any point to any straight line, and equal stretches be marked off in opposite directions from the foot of the normal along the straight line, then the lines joining their opposite extremities to the external point are congruent. This enables him to deal with the congruence of stretches on non-parallel lines. The only further assumption that is needed is that the velocity of $S_1$ in the timeless space of $S_2$ is equal and opposite to that of $S_2$ in the timeless space of $S_1$. With these assumptions he deduces the standard transformations of the Special Theory, except that where $c$, the velocity of light, appears in the latter, Whitehead has an undetermined constant $k$, which expresses the relation between the units in which we measure time and the units in which we measure length. It is simply an empirical fact that this constant $k$ is approximately identical in value with the velocity of light. It will thus be seen that Whitehead's deduction involves very much more fundamental considerations than the standard deduction. He ascribes three special merits to his method: (1) He has defined parallelism. (2) He has defined normality. (3) The notion of time-systems has given a clear meaning to the notion of a Newtonian frame of reference, and has solved the old philosophical difficulties about absolute rotation.

It is greatly to be wished that Whitehead had entered more into detail about this last claim. He has now made it in an incidental paragraph three times over in successive works. If it be true, it is of the utmost importance, and it ought to be discussed in some detail. At present I cannot see that Whitehead has accomplished anything more in this matter than the old doctrine of absolute Space and Time. Two questions can be raised about Newtonian frames. (1) What do we *mean* by them? and (2) How can we tell in practice when we have got hold of one? The old theory of absolute Space and Time gave a clear answer to the first and no answer to the second. We had in practice to take the fixed stars for our spatial axes and suitably adjusted pendulum clocks for our temporal standards. Whitehead's theory seems to me to be in exactly the same position. It gives an alternative answer to the first question, which is no doubt better in accord with the facts on which the Theory of Relativity is based. But, as regards the second, it seems to me to leave us with the fixed stars and pendulum clocks as our only means of determining what is in fact a time-system in Whitehead's sense. Of course, I do not suppose that he would make such claims unless he had good reasons for doing so. But I do know that much more competent mathematicians than I have failed to understand his precise point, and it therefore probably does need further elucidation.

The second and still more fundamental difference between Whitehead and the orthodox Relativists concerns the structure of Space-Time. The orthodox interpretation of the General Theory of Relativity is that Space-Time is non-homaloidal, *i.e.*, that it has not an uniform structure always and everywhere but that its structure varies wtth its contents. This whole book is a protest against this view. Whitehead is concerned to prove two points: (1) That Space-Time is and must be homaloidal; (2) That nevertheless the traditional law of gravitation can and must be modified, and that the modifications will account for the facts, such as the change in the perihelion of Mercury and the deflexion of light by the sun, which Einstein's theory accounts for. I will say a little about his views on these two points in turn.

(1) Whitehead has two main arguments in favour of the view that Space-Time must be homaloidal. His first argument may be called epistemological, for it practically amounts to saying that if Space-Time were non-homaloidal, induction would be impossible. His second argument has to do with measurement. So far as I understand it it asserts that the approximate agreements between the spatial measurements and between the temporal measurements of different people is only explicable on the assumption of the uniformity of Space-Time.

(*a*) The epistemological argument occurs in chap. ii., pp. 14-16 and in chap. iv., pp. 62-66. The first treatment is fuller, but perhaps the second is clearer. Whitehead distinguishes between "fact" and "factors". By "fact" he seems to mean something very much like the presentation-continuum of the psychologists, subject to the modification that it is not supposed to be mental or to be confined to one individual. "Factors" seem to mean limitations within fact, much as sensa are differentiations within a sense-continuum; but Whitehead apparently regards universals as factors, as well as the finite events in which these universals inhere. We must not regard factors as primitive, and fact as built out of them as a wall is built of bricks. On the other hand, we must not regard factors as differentiations of fact made by our own subjective activities. It is of the nature of fact to be differentiated into, though never exhausted by, factors. But it is also of the nature of any factor to have such and such relations to other factors of fact. *E.g.*, the factor red essentially involves a reference to some place which is red and to some time throughout which this place is red. Now we can know factors in two ways, *viz.*, "by adjective" and "by relation". When I see a red spot and note its redness I know it by adjective. But, as the spot has to be somewhere and somewhen in the whole which we call "nature," we also know it by its relations to those other factors in nature which place and date it. Many factors are known only by relation, I know that there is an inside to a box, though I may know nothing about the adjectives which qualify this inside. I know the inside by its spatial relations to the outside, which I know by adjective. This distinction between the two kinds of knowledge is obviously a good deal like that between acquaintance and description, though I doubt whether it exactly agrees with the latter antithesis.

Now, so far as I can understand Whitehead's argument for the uniformity of Space-Time, it runs as follows. I cannot know a factor without knowing other factors to which it is related. But I must not need to know *all* its relations to *all* other factors, or I should be unable to know anything till I knew everything. Whitehead therefore holds that we must distinguish between necessary and contingent relations. We must hold that there are certain relations to all other factors, which a given factor must have in order to be the factor which it is. But there are other relations which might have been different though the given factor was what it is. Also we must be able to know what these necessary relations are; for we only know the other factors of fact, to which the given factor refers, as those which have such and such relations to the given factor. Now this is only possible if the relations in question are the same throughout the *whole* of Nature as they are within *limited regions* of Nature in which they can be directly observed. The relations in question are spatio-temporal; and thus Whitehead claims to have proved that Space-Time must be homaloidal, if knowledge of Nature is to be possible.

I must confess that I should be greatly surprised if so concrete a result can be reached from such abstract premises. Even if we accept the whole of the argument up to the last step, I doubt if we have any right to take that further step. It seems to me that the most that such an argument could prove is that there are *some* uniform relations. No doubt spatio-temporal relations would then be plausible candidates for the position, but that is as far as one could go. Moreover, suppose it be granted that spatio-temporal relations are the ones which must be uniform, the question might still be raised: "*How much* uniformity must they possess?" There is a certain amount of uniformity even in Einstein's non-homaloidal Space-Time. The "separation" is everywhere an homogeneous quadratic function of the differentials of the four parameters which determine a point-event. Would it not be perfectly compatible with the epistemological argument to say that only *this* amount of uniformity was necessary, and that the particular coefficients of these differentials is contingent and variable?

(*b*) Whitehead's other argument to prove that Space-Time must be homaloidal is contained in chapter iii. This is a most interesting chapter in which Whitehead deals with the general theory of congruence and equality. The earlier part of this chapter is quite general. It points out that you cannot talk of "equality" in the abstract; equality always involves the "matching" of two things, *i.e.*, the possession by both of them of a certain one member of a certain class of qualities. Whitehead calls the class of qualities which is presupposed in any given case of equality "the qualifying class" and the class of things which have one or other of these qualities, "the qualified class". The minimum intelligible statement that can be made about the equality of two terms A and B is that "A has equality to B with respect to the qualifying class $\gamma$". This might be more shortly put in the form "A has $\gamma$-equality with B". It is quite evident that A might have $\gamma$-equality with B and might not have $\delta$-equality with B. *E.g.*, a pillar-box and a fire-engine have (roughly) colour-equality, but they do not have shape-equality.

He now applies these general considerations to the particular case of the congruence of stretches. He first takes a single unbounded straight line. The qualified class will now be the class of all finite stretches on this line. The qualifying class must be some set of qualities such that every stretch has one of them and no stretch has more than one. Two stretches will be congruent when they both possess one single quality from this set. He then lays down five conditions which the qualities in a qualifying class must obey if the "matching" of two stretches in respect to such qualities is to be what we understand by congruence.

So far Whitehead has simply been laying down a set of perfectly general conditions for the possibility of measurement. He now points out that in theory there might be innumerable different qualifying classes, all of which fulfil all these conditions. Indeed, I understand him to say that it can be proved from Lie's theory of Continuous Groups that this *must* be so. Now two stretches which are congruent with respect to one of these qualifying classes will be wildly incongruent with respect to others of them. So the problem at once arises: "Why do we all agree approximately as to what is congruent with what? Why are there not perfectly rational people who measure in a perfectly consistent way and make the distance from London to Glasgow shorter than the distance from London to Brighton?" His answer is that there must in fact be some one qualifying class which we all use in our judgments of congruence. This involves the recognition by us all of a certain fundamental spatio-temporal structure in Nature. It must be spatio-*temporal*; for we have to account for our approximate agreement about time-congruence, as well as for our approximate agreement about space-congruence. He further argues that this structure is due to the existence of different time-systems in Nature, such that the successive momentary spaces of any one of them have Euclidean parallelism to each other, whilst the momentary spaces of any two of them intersect each other.

It can hardly be doubted that Whitehead has here got hold of very important facts. It seems to me that he rather confuses the discussion for the lay reader by using these facts in quick succession for four purposes. First, he uses them to prove that the classical theory of measurement as developed by pure mathematicians is inadequate to account for all the facts. So far as a non-expert can judge, he seems to have made out a very good case for this. Secondly, he uses them to prove that the classical Relationist theory that Space consists of relations between *bodies* will not do, and must be replaced by a theory of spatio-*temporal* relations between *events*. This is undoubtedly true, though I do not think that its precise connexion with the present argument is made any too clear. Thirdly, he uses them to support the Special Theory of Relativity, as supplying just that stratification of Nature into different time-systems which provides the spatio-temporal structure needed to account for the agreement between our judgments of congruence. Lastly, I understand that he also uses them to show that the structure of Space-Time must be homaloidal, and cannot be such as Einstein has suggested in the General Theory of Relativity. Now it may be that there is a much closer logical connexion between all these apparently different conclusions than I can detect; if so, it is to be wished that Whitehead had explained it more fully. It is certainly not clear to me that the last conclusion follows at all. I should have thought that the *de facto* agreement of our judgments of congruence, so far as it goes, required nothing more than an approximately uniform spatio-temporal structure within those regions of extension and duration which we have measured. So far as I can understand, this condition would be fulfilled on Einstein's theory, since the departure from Euclidean structure is excessively small even in the neighbourhood of a huge gravitating mass like the sun.

I am therefore not persuaded that Whitehead has proved his main contention that Space-Time *must* be homaloidal. But the whole subject is so difficult and moves in such very unfamiliar regions that I should hesitate to express a positive opinion. Reviewers too often rush in where philosophers and mathematicians fear to tread. What seems to me certain is that Whitehead has produced important arguments which should make us pause before deserting the traditional views so far as to make Space-Time non-homaloidal. In addition to this he seems to me to have shown quite conclusively that there is nothing to *force* us to a non-homaloidal theory. He has succeeded in giving a modified law of gravitation which will do all that is needed of it, and which requires only the homaloidal Space-Time of the Special Theory of Relativity. I will end the review with a short sketch of Whitehead's suggested law.

(2) It would be quite impossible to enter in detail into Whitehead's suggested law. The most I can do is to state in outline and in my own words how it seems to me to be connected with the traditional law. Although I think I understand the general drift of Whitehead's argument from pages 71 to 82, I find some of the details extremely hard to follow. In fact the vitally important equations (9) and (13) seem to be shot out of a pistol, and Whitehead cannot expect to be understood by any reasonable number of people if he will not supply a little more help to his readers at the turning points of the argument. It is therefore possible that the account which I am going to offer is quite wide of the mark; the intending student of the book must therefore take my remarks for what (if anything) they are worth.

The traditional law calculates the gravitational potential at any point P in the following way. It takes a typical external point X, supposed to be occupied by a particle of mass $m_x$, and it says that the gravitational potential at P, due to this, is $\frac{\gamma m_x}{PX}$, where $\gamma$ is the gravitational constant and PX is the distance from X to P. To find the total gravitational potential at P it sums up for all external points such as X, getting $\sum_x \frac{\gamma m_x}{PX}$ as a result. To find the effective gravitational potential at P we must multiply this by $m_P$, the mass of the particle at P.

Now it is quite certain that this is not a permissible law of Nature on the view of Space-Time which Whitehead and the Special Theory of Relativity agree in accepting. A true law would be co-variant as between all the different equally permissible time-systems; but the traditional law, which is wholly in terms of spatial separation and makes no mention of temporal separation, cannot be so. On the other hand, we know that the traditional law is *numerically* very nearly true. Thus the problem is to find a law which shall lead to the traditional law as a first approximation, but shall be of such a form as to be covariant for transformations from any one time-system and its associated timeless space to any other. Such a law will certainly have to be in terms of *events* and their spatio-*temporal* separations, and not, like the traditional law, in terms of *bodies* and their purely *spatial* separations. To put it crudely, you must not consider what is going on at other places at the *same* moment in order to find the gravitational potential at a given place now. You must consider what *was* going on at any place at such a moment in the past that its influence would reach the given place at the present time. The further a place is from the given place the more remote is the event in its history which is relevant to what is going on in the given place now. This conception is quite familiar in electro-magnetics under the name of "retarded potentials," and it seems to me that the essence of Whitehead's modification is the introduction of retarded potentials into gravitational theory.

So far as I can understand, this is not quite the whole of Whitehead's modification of the traditional law. The other important point is this. The notion of a mass-point-event, *i.e.*, the occupation of a geometrical point at an instant by a mass, is obviously a product of extreme abstraction. Now I understand the other essential feature in Whitehead's view to be that this degree of abstraction is too extreme for dealing with the laws of physics. Instead of taking a point-event and considering the influence of all other correlated point-events and their contents on the nature of *its* contents, he finds it necessary to deal with slightly less abstract elements. He still keeps the spatial dimensions null (though he admits that this is probably an over-simplification); but he takes short historical stretches instead of momentary event-particles. Thus his problem is to find the influence which is exerted on the contents of an infinitesimal slice of the history of a point by the contents of infinitesimal slices of the history of all other points starting from correlated times. Thus his law of gravitation appears as an equation involving infinitesimal stretches along historical routes. This is perhaps the most original and philosophically interesting feature of his theory.

It would be impossible to over-estimate the importance and interest of this book. It shows Whitehead's powers of original thought and detailed mathematical application at their highest. But I am very much afraid that it will not have the influence which it ought to have. From the nature of its subject-matter it cannot be easy, and I cannot but think that it has been made quite needlessly difficult by excessive condensation. No doubt a professional mathematician would easily fill in certain gaps in the argument which a mere amateur like myself finds disconcerting. But I am at least fairly familiar with the subjects which it treats, and I am afraid that readers who do not start with that initial advantage will sometimes be tempted to give up in despair.

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